

Extracts from a Scrap Found in The Library of Babel¹

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[Hardy] And now, I said, let me show in a figure how far our nature is enlightened or unenlightened...

[Hirsch] Come now, Hardy, that’s Plato, Book VII of the Republic. We all know that. Or well we would, if cultural literacy had the pride of place that it should.

[Hardy] Hrm, yes, sorry. I’m new to this whole “Socratic dialogue” thing, so I thought I’d start with a classic, to sort of ease in.

[Hirsch] New to Socratic dialogue? Was it not you who wrote *A Mathematician’s Apology*, a defense of mathematics as stirring as Plato’s own, admittedly more widely read, *Apology*?

[Hardy] Well, that’s very kind of you. I was indeed the author of that text. But it was no dialogue – though it was, as you say, a defence.

[Hirsch] Defence?

[Hardy] Yes, with a “c.” Bloody Yanks and your shameless spelling errors. In any event, you’ve given me some momentum, so I thank you, and trust you to see yourself out.

[Hirsch] Oh?

[Hardy] With apologies. Our own interests don’t overlap so much, you see, so I think I will be dialoguing with some other luminaries.

[Hirsch] Fair enough. Happy to help. I’ll add only the following clarification. Your treatise is decidedly *not* one on mathematics education, or the philosophy of education. But it nonetheless has value to those interested in these subjects. As one of your earlier reviewers said, “This book is not only about mathematics, it is about ideals, art, beauty, importance, significance, seriousness, generality, depth, young men, old men and G. H. Hardy. It is a book to be read, thought about, talked about, criticized, and read again.”² I quite agree. Now, as you might say, “until the next time, old chap.”

¹ With thanks to J.L. Borges.

² Randolph, J. F. "Review of GH Hardy." *A Mathematician’s Apology*. *The American Mathematical Monthly* 49 (1942): 396-397.

[Hardy] I suppose I might as well read aloud sections of my *A Mathematician's Apology* and we'll see who pops in to discuss or dispute. Okay, §1. "The function of a mathematician is to do something, to prove new theorems, to add to mathematics, and not to talk about what he or other mathematicians have done. Statesmen despise publicists, painters despise art-critics, and physiologists, physicists, or mathematicians have usually similar feelings: there is no scorn more profound, or on the whole more justifiable, than that of the men who make for the men who explain. Exposition, criticism, appreciation, is work for second-rate minds."

[Mann] Whoa, Hardy! That's hardly fair.

[Hardy] How so? After all, perhaps you've heard the common saying, "Those who can, do; those who can't, teach."

[Mann] I haven't heard that, actually. I suppose it originated after my time. In any event, consider, as I note in my *Fourth Annual Report*, "The ability to acquire, and the ability to impart, are wholly different talents."

[Hardy] But surely they are related abilities?

[Mann] Not necessarily. "The former may exist in the most liberal measure, without the latter. It was a remark of Lord Bacon, that 'the art of well-delivering the knowledge we possess is among the secrets, left to be discovered by future generations.' Dr. Watts says, 'there are some very learned men, who know much themselves, but who have not the talent of communicating their knowledge.'"

[Hardy] Ah, true. Or perhaps they even have a talent for communicating their knowledge but dislike it all the same. In §29, near the end of my work, I acknowledge, "I hate 'teaching', and have had to do very little, such teaching as I have done been almost entirely supervision of research; I love lecturing, and have lectured a great deal to extremely able classes; and I have always had plenty of leisure for the researches which have been the one great permanent happiness of my life."

[Dewey] Interesting that you differentiate teaching from lecturing, Hardy. I wouldn't have expected a university professor to make such a distinction. At least you know what you like and don't like.

[Hardy] In any event, Mann, it sounds like I should distinguish two ideal categories: mathematicians, and teachers of mathematics. Now, I don't know whether those two sets ought to overlap, or whether one is contained in the other...

[Mann] Well, Alexander Pope noted, “Let such teach others who themselves excel.”

[Hardy] That seems like salutary advice. But I’m just not sure how to reconcile that with my feeling that those capable of “do[ing] something ... [of] add[ing] to mathematics” ought to be doing so.

[Mann] Well, don’t let it get you down. Your first chapter *is* rather gloomy. But speaking in defense of expositors, I’ll remind you of Euclid. His chief accomplishment, for which you mathematicians will forever be in his debt, was to order and arrange mathematics, to bring to the discipline, well, *discipline*. Even though he was one of those explainers, those consolidators that you decry, you still credit him as a pre-eminent mathematician. No second-rate mind, his!³

[Hardy] True, true. But see here, it’s about talent. I note in §3, “If a man has any genuine talent he should be ready to make almost any sacrifice in order to cultivate it to the full.”

[Locke] That’s very Aristotelian of you, Hardy. I assume, of course, you mean for your man of talent to cultivate his skills virtuously.

[Hardy] To be sure. I continue: “We have of course to take account of the differences in value between different activities. I would rather be a novelist or a painter than a statesman of similar rank; and there are many roads to fame which most of us would reject as actively pernicious. Yet it is seldom that such differences of value will turn the scale in a man’s choice of a career.”

[Locke] Yes, though plainly, virtue consists of more than just the rejection of the pernicious. In any event, a novelist or a painter being higher than a statesman? I suppose I could be expected to take issue with that, but I don’t. As I note in my dedication of *Some Thoughts Concerning Education*, we should educate in the fashion “which is the easiest, shortest, and likeliest to produce virtuous, useful, and able men in their distinct callings.” Distinct callings: each man may be disposed to a different occupation.

[Hardy] And being so dispo –

[Wollstonecraft] Wait a second, Locke, each *man*?

³ This defense of mathematics exposition in light of Hardy’s withering criticism, with specific reference to Euclid’s contributions, is due to Ben Orlin in the blog post “‘Explanation is work for second-rate minds,’ said the monster,” <<https://mathwithbad drawings.com/2016/05/11/the-second-rate-minds-strike-back-or-a-defense-of-explanation/>>, last visited 7/27/17.

[Locke] Now see here, Wollstonecraft, you know very well that “the principal aim of my discourse,” as I remarked in §6 of my work, was “how a young gentleman should be brought up from his infancy, which, in all things, will not so perfectly suit the education of *daughters*.”

[Wollstonecraft] I demand vindication! I dema—

[Hardy] Look, I’ll just stipulate right now, whether or not it’s true, that I’m perfectly fine with equal education for both sexes (even if you yourself, Wollstonecraft, weren’t advocating for exactly that). We can move on, okay?

[Locke] Okay.

[Wollstonecraft] Alright.

[Hardy] Where was I? Okay, in §7, “I am writing for readers who are full, or have in the past been full, of a proper spirit of ambition. A man’s first duty, a young man’s at any rate, is to be ambitious. Ambition is a noble passion which may legitimately take many forms; there was something noble in the ambitions of Attila or Napoleon; but the noblest ambition is that of leaving behind something of permanent value ... Ambition has been the driving force behind nearly all the best work of the world. In particular, practically all substantial contributions to human happiness have been made by ambitious men.”

[Rousseau] Oh, for god’s sake.

[James] No, I think he’s on to something! Ambition follows from the instinctive tendency of emulation, as I note in Chapter VII of my *Talks to Teachers on Psychology*. And emulation manifests itself in rivalry. “The feeling of rivalry lies at the very basis of our being, all social improvement being largely due to it.” Not just that sentimental rivalry claptrap with one’s former self that you talk about, Rousseau, but with others too. Indeed, one of your own critics noted, “the deepest stirring of action in us is the sight of action in another. The spectacle of effort is what awakens and sustains our own effort.”

[Hardy] Quite. Although I wonder if rivalry, as you describe it, is necessarily competitive, or if there’s a sort of cooperative rivalry too. After all, the two most valuable experiences of my professional life were my collaborations with Littlewood and Ramanujan, and those were not competitive endeavors.

[James] To be sure. Though I didn’t make this distinction in my *Talks to Teachers*, perhaps we could do so now: “rivalry,” let us say, refers to that emulous passion which is competitive, but “emulous passion”

as a category encompasses more than the merely competitive. Your work with Littlewood, with Ramanujan, it sprang from such a passion.

[Hardy] But on rivalry in children, James, you're also spot-on. I note in §29, on my boyhood, "I do not remember having felt, as a boy, any *passion* for mathematics, and such notions as I may have had of the career of a mathematician were far from noble. I thought of mathematics in terms of examinations and scholarships: I wanted to beat other boys, and this seemed to be the way in which I could do so most decisively."

[James] Exactly.

[Hardy] Back to the present, that passion in seeing and working with my peers – and I flatter myself in saying that on my best days, I was something like a peer of those great men – was not the first cause of my efforts, though, but the second. I continue in §7, "There are many highly respected motives which may lead men to prosecute research, but three which are much more important than the rest. The first (without which the rest must come to nothing) is intellectual curiosity, desire to know the truth. Then, professional pride, anxiety to be satisfied with one's performance, the shame that overcomes any self-respecting craftsman when his work is unworthy of his talent. Finally, ambition, desire for reputation, and the position, even the power or the money, which it brings." My intellectual curiosity, my hunger for rigor, came first.

[James] I'll grant you that. I also note that you distinguish pride from ambition, as did I. I said of pride and pugnacity, still in Chapter VII, that they are "most potent spurs to effort ... a general unwillingness to be beaten by any kind of difficulty what makes us feel 'stumped' and challenged by arduous achievements." Now, I didn't venture this far in my writing, but I suppose pride is an individual trait, while ambition is a social one. Pugnacity does not require other people, ambition does. Robinson Crusoe marooned on his island could have felt pride in his accomplishments but not ambition towards them. Now, certainly, pride and ambition are interconnected and hard to separate ... I decided to just call the whole group the Ambitious Impulses, but I think there's still some distinction there.

[Rousseau] Did someone say Crusoe?

[Locke] Never mind that, Rousseau, did you say something earlier about desire for reputation, Hardy? I think we're on the same page here. I note in §61 of *Some Thoughts* that reputation is "not the true principle and measure of virtue (for that is the knowledge of a man's duty and the satisfaction it is to

obey his Maker in following the dictates of that light God has given him with the hopes of acceptance and reward), yet it is that which comes nearest to it ... the testimony and applause that other people's reason, as it were by common consent, gives to virtuous and well-ordered actions."

[Hardy] I'm no fan of God, but we're of a like mind. My duty – as you say, the "dictates of that light" that I followed – was what I specified as my first motive earlier: creative intellectual discovery, the pursuit of mathematical truth. This was indeed "the true principle and measure of [my] virtue." Then, the pride that James and I were talking about just now, and lastly, the motive of reputation.

[Locke] You are, indeed, engaging in the "right improvement and exercise of [your] reason," which is "the highest perfection that a man can attain to in this life" (§122). But why ought pride come before reputation? I do not think highly of it. It's a short remove from obstinacy, which is the "one, and but one fault, for which I think children should be beaten" (§78).

[James] You're thinking of pride as stubbornness, disobedience, rebellion. The sense in which I used the word, and perhaps Hardy too, is nearer what you refer to as fortitude. You say of fortitude that it is "the guard and support of the other virtues ... the quiet possession of a man's self and an undisturbed doing his duty" (§115).

[Hardy] Yes. Pride, as I conceive of it, is ensuring that my work is worthy of my talent. And that, really, is nothing other than what you would describe as fortitude, which is to say, strength in the exercise of my reason (and other virtues).

[Locke] Ah. So your first two motives accord with my sense of the ideal of man as he who exercises his reason. And your third motive corresponds to my notion of reputation.

[James] Well, actually, Hardy, maybe you and I are not exactly on the same page. For me, pride is a fundamental psychological instinct. It exists prior to rational man.

[Hardy] So?

[James] Well, in that sense, it's not like fortitude. As you're now representing it, your pride, being fortitude, is a rational act, not the instinct that I've identified it as.

[Hardy] Perhaps it could be both: an instinct harnessed for rational purposes. Or my pride, your pride, and Locke's fortitude are not identical categories after all. Or perhaps, James, I can split the difference between you and Locke. After all, I say that out of pride I want my work to be worthy of my talent – i.e. I

want to be the fortitudinous Lockean reasoning man – but I also couch that same desire in terms of anxiety and shame, and there’s your psychology, your instinctual passion.

[James] Hmm. Okay. Let’s move on.

[Mann] I want to press you a bit: why *be* educated?

[Hardy] Well, as Hirsch commented earlier, this is not a subject I take on directly. And I’m afraid what I have to say is neither too complementary nor too democratic. But let’s start by talking about utility. In §19, “A science or an art may be said to be ‘useful’ if its development increases, even indirectly, the material well-being and comfort of men, if it promotes happiness, using that word in a crude and commonplace way. Thus medicine and physiology are useful because they relieve suffering, and engineering is useful because it helps us to build houses and bridges, and so to raise the standard of life (engineering, of course, does harm as well, but that is not the question at the moment).”

[Mann] So we should all study what’s useful? To try and promote commonplace happiness?

[Hardy] Not hardly. Consider §20. “Physiology and engineering are not useful studies for ordinary men (though their study may of course be defended on other grounds). For my own part I have never once found myself in a position where such scientific knowledge as I possess, outside pure mathematics, has brought me the slightest advantage. It is indeed rather astonishing how little practical value scientific knowledge has for ordinary men, how dull and commonplace such of it as has value is, and how its value seems almost to vary inversely to its reputed utility.”

[James] Okay, as a scientist, let me try to piece this out: you distinguish what is practical, what is useful for the lives of “ordinary men,” from what has advantageous scientific value. Based on what you say elsewhere in your work, this second category is an aesthetic and intellectual one, not a utilitarian one. Indeed, the thrust of your work is a defense of mathematics as an aesthetic exercise. In any event, education can allow us to develop along both lines.

[Hardy] Yes. Although, just because physiology as a discipline is useful, “[o]f course we do not mean ... that most people ought to study physiology, but that the development of physiology by a handful of experts will increase the comfort of the majority” (§20).

[Mann] But isn’t it useful for everyone to be versant in these things?

[Hardy] No. Consider, “[w]e know that the gas will burn without knowing its constitution; when our cars break down we take them to a garage; when our stomach is out of order, we go to a doctor or a drugstore. We live either by rule of thumb or on other people’s professional knowledge” (§20).

[Mann] I think that’s shortsighted. You may be right that that’s how we live, but your “handful of experts” won’t select themselves. Students can only identify interests if they’re exposed to them in the first place.

[Hardy] Nonetheless, “this is a side issue, a matter of pedagogy, interesting only to schoolmasters who have to advise parents clamouring for a ‘useful’ education for their sons” (§20).

[Mann] It may be a side issue to you, but it’s of paramount importance to me, and the rest of the country! Okay, well, you’re English and I’m American, but you know what I mean. Common education is our best means of removing poverty and securing national abundance; of, in our democratic society, ensuring better self-government; and most fundamentally, of elevating the individual and the state.

[Dewey] Precisely. And, as in articles I and II of *My Pedagogic Creed*, also to teach the child “to act as a member of a unity,” to enable him “to share in the inherited resources of the race, and to use his own powers for social ends.”

{At this point, a squabble of other philosophers and educators pile on, discoursing on the virtues and importance of education.}

[Hardy] Okay, okay, perhaps I was overly dismissive. Remember, I only ever set out to write about mathematics and the performance thereof. As for my various offhand comments, I might have fleshed out or argued them differently in different circumstances. And I’ll acknowledge that, say, “school” mathematics, “which we can call ‘social’ if we please,” does have many uses, and is worth knowing and being taught (§27). I mean simply to say, on aesthetic grounds, this “school” mathematics is far inferior to “real” mathematics. That it’s more practical or useful is irrelevant.

[Russell] Hardy, my good man! I quite agree with where you’re coming from. Take what I say in *The Negative Theory of Education*. “Three divergent theories of education all have their advocates in the present day. Of these the first considers that the sole purpose of education is to provide opportunities of growth and to remove hampering influences. The second holds that the purpose of education is to give culture to the individual and to develop his capacities to the utmost. The third holds that education is to be considered rather in relation to the community than in relation to the individual, and that its

business is to train useful citizens.” To the extent that you yourself describe education, you align most closely with the second theory in what I indicated just now.

[Hardy] Yes, broadly speaking. However, your “culture to the individual” means, to me, ideas, thoughts, mathematical and scientific structures –intellectual inheritance, rather than simply cultural. And it’s not just an individual inheritance. It is to be developed both individually and in the context of a like-minded community.

[Bestor] This second theory of Russell’s, your theory, Hardy, and my own, all seem to align. (Again, with the caveat that you, Hardy, rarely talk about education as such.) As I note in *The Distinctive Function of the Schools*, societies pursue three aims in developing children to adults: instructing in practical skills, relating the mores of a society, and training “in the use [and development] of the intellectual tools that the civilization has developed.” I think that a school qua school should be concerned with only the last of these three. And you too, it seems, perceive that the most worthwhile endeavor in life, after maintaining some base level of material comfort, is intellectual exploration, which is what I want to have schools for.

[Hardy] Certainly that’s how I present it in *A Mathematician’s Apology*. “When the world is mad, a mathematician may find in mathematics an incomparable anodyne. For mathematics is, of all the arts and sciences, the most austere and the most remote, and a mathematician should be of all men the one who can most easily take refuge where, as Bertrand Russell says, ‘one at least of our nobler impulses can best escape from the dreary exile of the actual world’” (§28).

[Russell] Hey, thanks for the citation!

[Hardy] One last piece of support, if I may, for my intellectual/aesthetic tendencies. I say, “I have just one chance of escaping a verdict of complete triviality, that I may be judged to have created something worth creating. And that I have created is undeniable: the question is about its value. The case for my life, then, or for that of any one else who has been a mathematician in the same sense which I have been one, is this: that I have added something to knowledge, and helped others to add more” (§29).

[James] Okay, I suppose I’m not surprised to find that you, Russell, and Bestor all adopt a pretty rationalistic, intellectual view of education and life. And this is contra that of many other philosophers of education, not least myself, as in Chapter III of my *Talks to Teachers* : “Whatever of transmundane metaphysical insight or of practically inapplicable aesthetic perception or ethical sentiment we may

carry in our interiors might at this rate be regarded as only part of the incidental excess of function ... man, whatever else he may be, is primarily a practical being, whose mind is given him to aid in adapting him to this world's life." Well, to each his own. But "verdict of complete triviality," Hardy, really? That's harsh.

[Snow] I might step in here, James. Of all those who've spoken here, only Russell and myself actually knew Hardy, after all. Hardy, allow me to share something I wrote of your work some two decades after you died, in my *Variety of Men* biographical collection. I find your *Apology* "a book of such haunting sadness. Yes, it is witty and sharp with intellectual high spirits: yes, the crystalline clarity and candor are still there: yes, it is the testament of a creative artist. But it is also, in an understated stoical fashion, a passionate lament for creative powers that used to be and that will never come again."

[Hardy] I suppose that's on the nose, Snow. And very well put, too. Thank you.

[Snow] No, thank you. I hope you find solace in, not only all the beautiful mathematics that you created, but also in the value that such a lucid and – if I may slander you so, given what you said earlier – expository work as *A Mathematician's Apology* will provide for generations to come.

[Hardy] Harrumph. Well, gentlemen – Ms. Wollstonecraft – thank you all for joining me. If I may say so, this has been a most edifying experience. But more pressing matters call: I've an appointment now with Ramanujan. He and I started the most intriguing problem on mock theta functions yesterday...